Propulsion Dynamics of Lunar Hoppers

RONALD J. MEETIN*
Rocket Propulsion Laboratory, Edwards Air Force Base, Calif.
AND
HOWARD S. SEIFERT†
Stanford University, Stanford, Calif.

A feasibility study was recently completed on a hopping transporter concept for locomotion on the moon. Termed a Lunar Pogo because it would operate similarly to a conventional pogo stick, this vehicle would accelerate up an inclined leg, pick up the leg, and then enter ballistic flight over the moon's surface. Upon recontacting the lunar surface, the Lunar Pogo would decelerate down the leg. Propulsion would be provided by expansion of gas against a piston. Operation would be partially conservative because much of the energy expended by the gas during takeoff would be recovered by compressing the gas during landing. Two models of the ballistics and propulsion have been set up to estimate performance. The simplified first-order model illuminates the thermodynamics of ideal operation. The second-order model, which includes realistic effects such as sliding, provides a better approximation to actual performance. In a one-man Lunar Pogo, an astronaut would typically make 15-m (50-ft) leaps at an average speed of 5–8 km (3–5 miles) per hour.

Nomenclature

A = cross-sectional area of piston face
E = internal energy
M = mass of main body
P = cylinder gas pressure
P_{acc} = accumulator gas pressure
R = gas constant
T = cylinder gas temperature
T_{acc} = accumulator gas temperature
V = cylinder volume
X_H = horizontal range
X_R = $X_H-(2)^{1/2}r_{Mo}$
c_v = specific heat at constant volume
d = cylinder displacement
d_M = distance from forward end of cylinder to main body center of mass
g = acceleration of lunar gravity
l_1 = length of leg
m = m_1+m_F
m_F = mass of foot
m_g = mass of cylinder gas
m_1 = mass of leg
r_M = l_1+d_M+d = radial distance from pivot to center of mass of main body
t = time
α = launch angle
γ = specific heat ratio
μ_{kin} = kinetic coefficient of friction
μ_{stat} = static coefficient of friction
σ = average surface slope

Subscripts

c = command or desired
d = disengagement
e = engagement
f = final
o = initial
opt = optimum
$-$ = minus (limiting value approached from below)
$+$ = plus (limiting value approached from above)

Received April 8, 1974; revision received July 25, 1974. This work was supported by NASA under Grant NGR-05-020-258.
Index category: Extraterrestrial Surface Transportation.
* Captain, USAF; presently at Hastings College.
† Professor, Department of Aeronautics and Astronautics. Fellow AIAA.

Introduction

THE fact that the surface gravitational field of the moon is only one-sixth the terrestrial surface field has made the concept of lunar transportation by *hopping* intriguing. Preliminary discussion of this idea first appeared in 1967.[1] In 1971 a feasibility study was completed at Stanford University on this new means for locomotion.[2] A vehicle using this mode would operate similarly to a conventional pogo stick. Termed the Lunar Pogo, this vehicle could consist of a main body, thrust leg, and foot for takeoff and landing. Figure 1 shows a conceptual one-man Pogo schematic. The main body accelerates up the inclined leg, picks up the leg and foot, and then enters ballistic flight over the surface of the moon. During free flight, the leg is rotated toward the direction of the touchdown velocity vector. When the foot recontacts the lunar surface, the main body decelerates down the leg. Studies performed concurrently with this one on propulsion show that the device could be attitude controlled gyroscopically during flight,[3] and

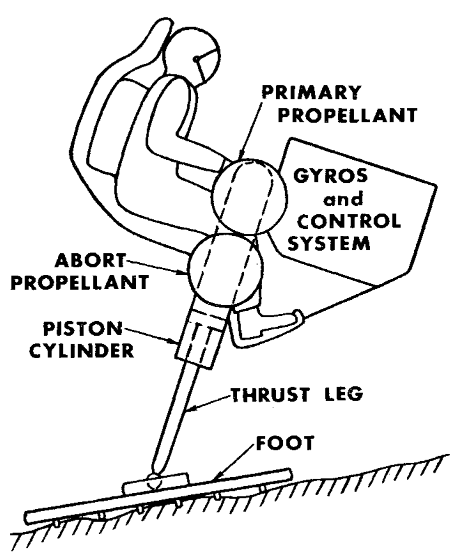

Fig. 1 Artist's drawing of a one-man Lunar Pogo.

that the accelerations which occur during the hopping process could, by proper design, probably be made tolerable to a human pilot.[4]

The Lunar Pogo would be capable of climbing and operating in moderately rough lunar terrain. In a one-man version, the pilot would typically make 15-m (50-ft) jumps at an average ground speed of 5–8 km (3–5 miles) per hour with a conservative operating range of 16 km (10 miles).

Ballistics and Propulsion

The propulsion unit for the Lunar Pogo imparts a specified velocity to the hopper at takeoff and, assisted by the control system, brings the Pogo safely to zero velocity at landing. The design criteria for the propulsion unit are that it operate for a given set of hop ranges, 0–30 m (0–100 ft), and slopes, $-20°$ to $20°$, tolerate a small amount of sliding, produce an acceleration profile acceptable to the pilot, be of as low weight as possible, minimize fuel consumption, be reliable, and be easily controllable from hop to hop. The ballistics for the Pogo includes the in-flight motion from lift-off to touchdown as well as the on-ground motion between hops. Since the ballistics and propulsion are closely related, the two areas have been analyzed together.

To assess performance, two models of the ballistics and propulsion for the Pogo have been set up and analyzed. The simplified first-order model provides the thermodynamics for ideal operation. The second-order model, which includes realistic effects such as sliding and friction, gives a better estimate for actual operation.

The following discussion is only a brief summary of the ballistics and propulsion. Initial analysis was done by M. Kaplan.[5] Extension of this work is presented in the reports on NASA Grant NGR-05-020-258, "Small Scale Lunar Surface Personnel Transporter Employing the Hopping Mode."[2,6–8] A more complete analysis of the work done to date is given in Ref. 9, from which this paper is largely extracted.

In support of the mathematical analysis, a 25-kg (55-lbm) terrestrial demonstrator was designed and constructed to show experimental feasibility of hopping locomotion. Shown in Fig. 2, this demonstrator could execute approximately 20 hops at 4 m (12 ft) per jump. The terrestrial demonstrator propulsion system is described in Refs. 7 and 8, while details on the structure are presented by S. Peterson in Ref. 10.

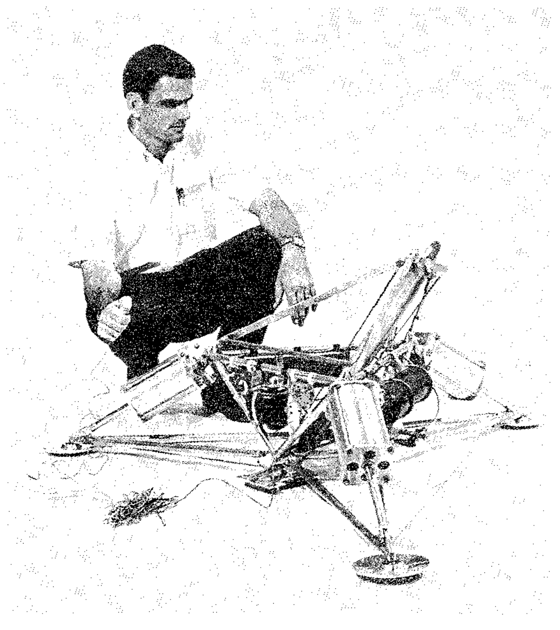

Fig. 2 Terrestrial demonstrator for hopping locomotion.

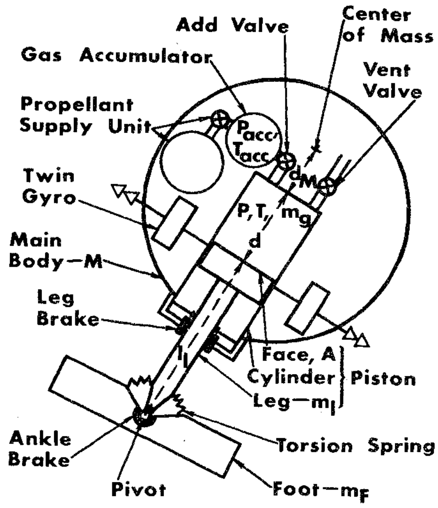

Fig. 3 Propulsion components for a Lunar Pogo.

Propulsion Design

Several mechanical methods were considered for the primary propulsion. For low weight, a gas-piston system was selected over a metal-spring system. For simplicity, a decision was made to concentrate on a direct-drive system in which the piston is directly attached to the leg instead of a geared or levered system in which a small piston displacement is magnified to produce a large main body displacement.[6,9]

Figure 3 shows the important propulsion components for a typical Pogo. The accumulator provides high-pressure gas to the piston cylinder. Gas is vented from the cylinder to the lunar vacuum. Normal pressure adjustments are made by adding gas to or venting gas from the cylinder. The propellant supply unit includes propellant tanks, valves, and a feed system. The leg brake controls (decreases) the relative velocity between the leg and main body. The ankle brake at the pivot limits the relative angular velocity between the foot and leg. Attitude control during flight is provided by two programed counter-rotating twin gyros. Since the attitude control system does not affect the trajectory of the center of mass, it is not discussed in this paper, although the attitude control system is obviously of vital importance. Details of experimental and theoretical work on attitude control are given by S. Pasternack in Ref. 3.

In normal operation the Pogo is initially on the ground with the leg oriented vertically. The pilot sets the desired launch direction, horizontal range, vertical climb, and launch angle into the system to initiate the hop. The Pogo rotates about the vertical to the launch direction and then tips to the launch angle. The cylinder pressure is adjusted to a predicted initial value. After the ankle brake is locked and other takeoff preparations are completed, the leg brake is released. The gas in the cylinder expands, forcing the main body to accelerate up the leg. The foot may or may not slide along the ground during acceleration. When a specified main body velocity/cylinder displacement profile is reached, the leg brake engages or picks up the leg as the Pogo enters free flight.

During free flight, the Pogo reorients while following a ballistic parabola. Free flight ends as a part of the foot touches down on the ground. The foot then "slaps down" toward the ground. During free flight or slapdown, the cylinder pressure may be readjusted. As the entire foot impacts the ground, the leg brake releases to disengage the leg from the main body. The main body decelerates down the leg, recompressing the gas. Simultaneously, the Pogo rotates toward the vertical. Again, the foot may or may not slide. When the relative velocity between the main body and leg reaches zero, the leg brake locks. Shortly afterwards the ankle brake locks as the Pogo reaches the vertical to end the hop.[2,8,9]

Propulsion operation is partially conservative because much of the energy expended by the gas during acceleration is recovered at deceleration. However, pressure adjustments in the cylinder through adding and venting must generally be accomplished to account for energy losses (e.g., engagement) and elevation changes. When these adjustments are made during the propulsion cycle depends on the sophistication of the propulsion and control systems. Specific propulsion operational modes for adjusting cylinder pressure are discussed in Ref. 9.

Either a cold (nonreacting) gas or a hot (reacting) gas could be used in the propulsion unit. A cold gas system would operate near room temperature. Possible cold gases include air, He, H_2, NO, N_2, and O_2. Since nitrogen is intermediate in properties related to propulsion among the cold gases, a cold gas system using N_2 was chosen for analysis. However, it is not yet clear which (if any) of the cold gases offer the best over-all performance. A hot gas system, operating near 2000°R, might use hydrazine (N_2H_4), which decomposes catalytically or thermally into gaseous ammonia, nitrogen, and hydrogen.

For equivalent missions a hot gas N_2H_4 system will consume considerably less propellant than a cold gas system. The gas savings factor for a hydrazine system over a cold gas (liquified) nitrogen system has been estimated at five. However, a hydrazine system requires a catalytic reaction chamber in addition to an accumulator. While a cold gas system appears relatively simpler, there could be logistic problems in storing the cold gas in liquid form. No realistic estimates of the hardware weight for either system have yet been made. Consequently, the choice of gas for the primary propulsion is still open.[2,9]

First-Order Model

To estimate the ideal performance for the Pogo, a first-order model of the ballistics and propulsion has been set up.[5,6,9] This ideal model is a nonsliding, but otherwise frictionless, approximation to the Pogo. A simplified operational cycle is used to minimize interaction between the propulsion system and the rest of the hopper. There are six significant time intervals:

- $t_f \to t_o$: on-ground wait (between hops)
- $t_o \to t_{e-}$: acceleration (of main body up leg) } takeoff
- $t_{e-} \to t_{e+}$: engagement (of leg to main body) } takeoff
- $t_{e+} \to t_{d-}$: free flight
- $t_{d-} \to t_{d+}$: disengagement (of leg from main body) } landing
- $t_{d+} \to t_f$: deceleration (of main body down leg) } landing

Other assumptions for the first-order model are that: 1) the hopper motion is two dimensional; 2) the lunar surface is locally straight and incompressible at the takeoff and landing areas; 3) the cylinder gas is a perfect gas ($PV = m_g RT$) with constant specific heats ($E = m_g c_v T$, $\gamma =$ const); 4) there is no heat flow through cylinder walls (i.e., adiabatic process); 5) the equilibrium gas relations are valid in the dynamic situations; 6) angular motion during acceleration and deceleration can be neglected; 7) engagement occurs instantaneously at lift off; 8) the linear momentum of the system is conserved at engagement; 9) the foot touches down parallel to the local lunar surface; 10) disengagement occurs instantaneously at touchdown; 11) the foot/leg combination loses all its momentum to the ground at disengagement without affecting the main body; 12) there is no gas transfer during acceleration or deceleration; and 13) isentropic expansion during acceleration and compression during deceleration occur ($PV^\gamma =$ const).

From these assumptions, the first-order equations of motion for acceleration, free flight, and deceleration were derived.[5,9] The free-flight equations can be integrated completely. Using the integrated free-flight equations, the optimum launch angle which maximizes the range is $\alpha_{opt} = 45° + \sigma/2$. ($\sigma$ is the average surface slope.) The equations for acceleration and deceleration can each be analytically integrated once. Using the entire set of integrated equations, the cylinder pressures can be determined as a function of the range, surface slope, cylinder displacements, and launch angle. Specializing to α_{opt} for $-10° \leq \sigma \leq 10°$ with a numerical error of less than 1%, the initial cylinder pressure P_o is[6,9]:

$$P_o \simeq \frac{(\gamma-1)(M+2m)g}{2A[1-(d_o/d_e)^{(\gamma-1)}]}\left(\frac{X_R}{d_o}\right)\tan\alpha_{opt} \tag{1}$$

and the final cylinder pressure P_f is[6,9]:

$$P_f \simeq \frac{(\gamma-1)Mg}{2A[1-(d_f/d_d)^{(\gamma-1)}]}\left(\frac{X_R}{d_f}\right)\cot\alpha_{opt} \tag{2}$$

where $X_R = X_H - (2)^{1/2} r_{Mo}$.

Equations for the adding and venting processes have also been derived.[9] To describe the thermodynamic cycle of the cylinder gas for a hop, these equations were combined with the equations for P_o and P_f and with other relations derived from $PV^\gamma =$ const and $PV = m_g RT$. By specifying the lunar terrain slope, horizontal ranges, cylinder displacements, and launch angles for a series of hops, the long-term effects on properties of the cylinder gas in the ideal model were studied.[2,9]

Figure 4 shows the thermodynamic variations in cylinder pressure and gas mass for a typical hopping sequence. The level, uphill, and downhill pattern is repeated indefinitely. The Pogo parameters for this sequence are shown in Table 1

Table 1 Pogo parameters for first-order model

$M = 540$ kg, 37 slug	$X_H = 15$ m, 50 ft
$m = 44$ kg, 3 slug	$d_o = d_f = 0.3$ m, 1 ft
$1_1 = 1.2$ m, 4.0 ft	$d_e = d_d = 0.9$ m, 3 ft
$d_M = -0.15$ m, -0.5 ft	$\gamma = \gamma(N_2) = 1.40$
$A = 183$ cm², 28.3 in²	$R = R(N_2) = 297$ m-nt/kg-°K, 1775 ft-lb/slug-°R
$T_{acc} = 294$°K, 530°R	$g = 1.62$ m/sec², 5.31 ft/sec²

where $r_{Mo} = 1_1 + d_M + d_o$.

The solid lines in Fig. 4 show the "equilibrium" thermodynamic cycle (for three-hop cycles) approached as the number of hops in the sequence becomes very large. The dotted lines show the variations in gas mass for three hops early in the sequence. Every third hop in this sequence has identical pressure, temperature, and gas mass variations at the "equilibrium" hopping conditions. The "equilibrium" cycle is reached when the average energies of the gas entering and leaving the cylinder equalize. For an arbitrary series of hops, the first-order model will still attain "equilibrium" hopping conditions. There will be no definite repetitive pressure, temperature, or gas mass variation patterns for an arbitrary sequence. However, the "equilibrium" hopping conditions will be marked by the stabilization of the mass of cylinder gas between two limits. One objective of the first-order model work was to estimate the ideal propellant consumption for a 16-km (10-mile) trip. The terrain shown in Fig. 4 is considered typical of the lunar surface. Using this

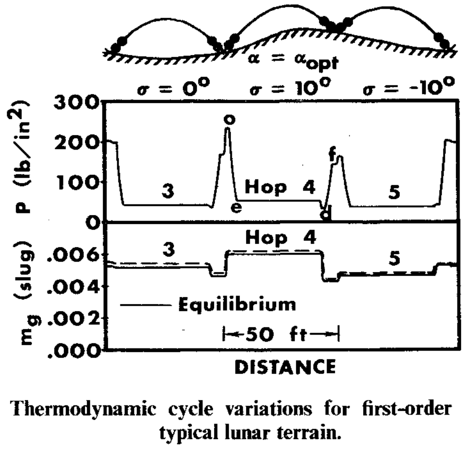

Fig. 4 Thermodynamic cycle variations for first-order model over typical lunar terrain.

sequence, the "equilibrium" consumption for 1000 hops at 15 m (50 ft) per hop is 12 kg (0.8 slug) of N_2. To this amount a gas allowance of 75–100% is added to allow for nonoptimum launches, rougher terrains, refillings if necessary, ullage, and a safety reserve. The total N_2 required for the 10-mile trip is 22 kg (1.5 slug). The corresponding amount of N_2H_4 is 4 kg (0.3 slug).[2,9]

A number of tradeoffs for optimizing performance have evolved from the first-order model. For example, operating at $\alpha > \alpha_{opt}$ to prevent sliding will decrease the average surface speed or increase the propellant consumption. The performance loss can be severe for small departures from α_{opt}. If launch is at $\alpha = 60°$ with $\alpha_{opt} = 45°$, the 15° increase over optimum will lead to a 29% loss in surface speed during free flight. Further tradeoffs are presented in Ref. 9.

Second-Order Model

To obtain a more realistic estimate of the Pogo performance, an advanced or second-order model of the ballistics and propulsion has been set up.[8,9] Although satisfying the first five assumptions listed for the first-order model, the advanced model includes realistic effects such as sliding between the foot and ground, friction at the pivot and in the piston, rotation during deceleration, and noninstantaneous engagement. The effects of the leg brake, ankle brake, propellant-feed system, gyro-control system,‡ foot-control mechanism, and prediction model for cylinder pressure are considered. Touchdown occurs with the foot nonparallel to the local lunar surface, and with a small offset angle between the directions of the leg and system velocity vector to assist in rotation to the vertical. The second-order model represents the next step toward a realistic design.

In analyzing the second-order model, heuristic functional relationships were first used to describe the various system components and friction laws. The general differential equations were then derived for an entire hop. Although too numerous and complex to be discussed briefly here, they are available in Ref. 9. Appropriate physical parameters were next inserted into the functional relations. For example, sliding was described by a set of coefficients of friction. Since the resulting equations are highly nonlinear, they were organized into a computer program for numerical integration.

A cold gas system with the same basic features as the first-order model studied (i.e., M, m, 1_1, d_M, A, T_{acc}, γ, R, and g) was examined. Specific values for the control parameters and gyro characteristics were chosen to give good performance for 15-m (50-ft) leaps over 10° slopes. However, no attempt was made to optimize the system. Typical frictional forces were assumed. Characteristics for the cylinder valves were selected that would enable the pressure adjustments of up to 70 nt/cm^2 (100 lb/in^2) required for landing to be accomplished in several tens of milliseconds. A very large vent valve on the order of 6 cm^2 (1 in^2) cross-sectional flow areas was needed because the pressure differentials across the valve were often very small.

The solutions to about 80 assorted hops were computed for the ranges of hop parameters presented in Table 2.[9]

Table 2 Ranges of hop parameters for second-order model

Parameter	Range of parameter
Average surface slope	$-20° \leqq \sigma \leqq 20°$
Desired horizontal range	9 m (30 ft) $\leqq X_{Hc} \leqq$ 18 m (60 ft)
Static coefficient of friction	$0.67 \leqq \mu_{stat} \leqq 2.00$
Kinetic coefficient of friction	$0.38 \leqq \mu_{kin} \leqq 1.20$

Hops were investigated for desired launch angles of $\alpha_c = \alpha_{opt}$, 45°, and 60°. A few near-vertical (reconaissance) hops as well as several jumps over hills and valleys were calculated.

‡ The attitude control laws of Ref. 3 were used.

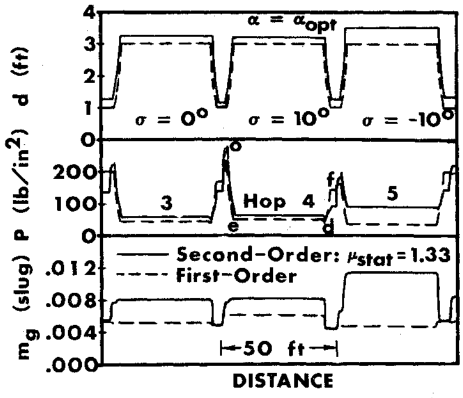

Fig. 5 Hopping cycle variations for first- and second-order models over typical lunar terrain.

Sliding during take off and landing was the most important phenomenon studied. Negative sliding or backsliding during launch increases with increasing uphill slope and decreasing coefficient of friction. For small amounts of backsliding, the control system normally compensates such that the horizontal range is negligibly shortened. However, as takeoff backsliding increases, drastic losses in range can occur. For example, beyond an initial 13 cm (5 in.) of backsliding in the system examined, the loss in range was about 30 cm ($2\frac{1}{2}$ ft) for each additional cm (inch) of sliding.

Although less critical than takeoff sliding, forward sliding at landing can greatly affect the final orientation of the Pogo. If landing sliding is severe, the Pogo may fail to reach the vertical position. Some technique must then be used to right the hopper. In addition, the large variety of landing conditions make sliding at landing less predictable than at launch.

To determine longer-term effects in the Pogo, several short sequences of hops were computed. Figure 5 shows the cycles for cylinder displacement, pressure, and gas mass for three hops of the first- and second-order models over a lunar terrain corresponding to that of Fig. 4. Based on analysis of these hopping sequences, "equilibrium" hopping conditions did not appear achievable in the system studied. The calculated propellant consumption was also several times that of the first-order Pogo. However, the second-order system was not optimized. The performance was decreased because of sliding and sluggish characteristics assumed for several of the Pogo components. Consequently, the data on this system should not be extrapolated. More information is needed on the actual lunar surface and on the real characteristics of likely Pogo components in order to better estimate propellant consumption and to predict the ability of the Lunar Pogo to reach "equilibrium" hopping conditions.[9]

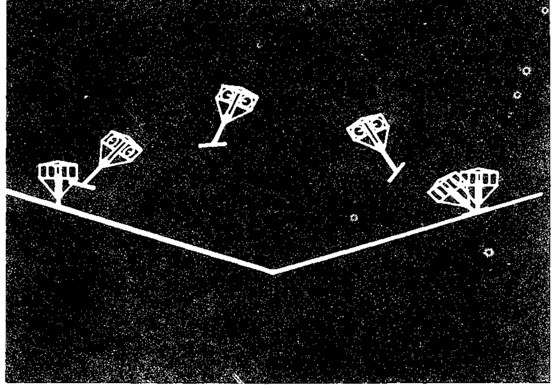

Fig. 6 Hopper flight simulation displayed on cathode ray screen.

To see real time hops, a computer-graphical display of the second-order model doing a hop was set up. In this display a schematic of the Pogo executes a jump on a cathode ray tube using computer solutions to the equations of motion. Figure 6 shows a multiple exposure of this display. Ten different hops were shown in this way. A short movie of these hops was made to complete the second-order analysis.[8,9]

Summary and Conclusions

This study has provided some basic design data for the development of the Lunar Pogo propulsion unit. A gas piston system using either a hot or a cold gas can supply the primary propulsion. Pressure adjustments for hop control will be made by adding gas to or venting gas from the cylinder.

Two models of the ballistics and propulsion were set up to obtain performance estimates. In the simplified first-order model, "equilibrium" hopping conditions were obtained in which the mass of cylinder gas stabilizes between two limits. Ideal propellant consumption is quite low. For a 16-km (10-mile) trip, the estimated cold gas N_2 consumption is about 4% of the total Pogo weight while the hot gas N_2H_4 consumption is less than 1% of the system weight.

The advanced second-order model demonstrated the need to minimize sliding. Takeoff sliding results in shorter hops. Severe sliding at landing can lead to undesirable final orientations. In addition, large fast-response cylinder valves, especially for venting, are needed to accomplish the required pressure adjustments. Although "equilibrium" hopping conditions did not always appear attainable in the second-order model, optimization could modify this conclusion.

In sum, the first-order analysis yields criteria for numerical selection of some of the important Pogo parameters and shows the thermodynamics of ideal operation. The second-order model highlights the major realistic effects and offers some estimates of their magnitudes. To obtain good performance, a rather sophisticated control system would probably be required for the Lunar Pogo.[2,9]

References

[1] Seifert, H. S., "The Lunar Pogo Stick," *Journal of Spacecraft and Rockets*, Vol. 4, No. 7, July 1967, pp. 941–943.

[2] Seifert, H. S., et al., "The Lunar Hopping Transporter," Final Report for NASA Grant NGR-05-020-258, SUDAAR 428, July 1971, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.

[3] Pasternack, S., Jr., "Attitude Control of Hopping Vehicles," Ph.D. dissertation, May 1971, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.

[4] Degner, R. L., "Effects of Cyclic Acceleration Pulses on Primates," Ph.D. dissertation, Sept. 1971, Mechanical Engineering Dept., Stanford Univ., Stanford, Calif.

[5] Kaplan, M. H. and Seifert, H. S., "Investigation of a Hopping Transporter Concept for Lunar Exploration," SUDAAR 348, June 1968, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.; also see *Journal of Spacecraft and Rockets*, Vol. 6, No. 8, Aug. 1969, pp. 917–922.

[6] Seifert, H. S., et al., "Small Scale Lunar Surface Personnel Transporter Employing the Hopping Mode," Semi-Annual Status Report of NASA Grant NGR-05-020-258, SUDAAR 359, Sept. 1968, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.

[7] Seifert, H. S., et al., "Small Scale Lunar Surface Personnel Transporter Employing the Hopping Mode," Annual Report of NASA Grant NGR-05-020-258, SUDAAR 377, May 1969, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.

[8] Seifert, H. S., et al., "Small Scale Lunar Surface Personnel Transporter Employing the Hopping Mode," Status Report of NASA Grant NGR-05-020-258, Period March 1, 1969–Sept. 1, 1970, SUDAAR 397, Sept. 1970, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.

[9] Meetin, R. J., "Ballistics and Propulsion for a Lunar Hopping Transporter," Ph.D. dissertation, May 1973, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, Calif.

[10] Peterson, S., "A Terrestrial Prototype of a Hopping Lunar Transporter," Engineer's thesis, Nov. 1971, Mechanical Engineering Dept., Stanford Univ., Stanford, Calif.